Topological Data Analysis for Trustworthy Artificial Intelligence

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1. Overview

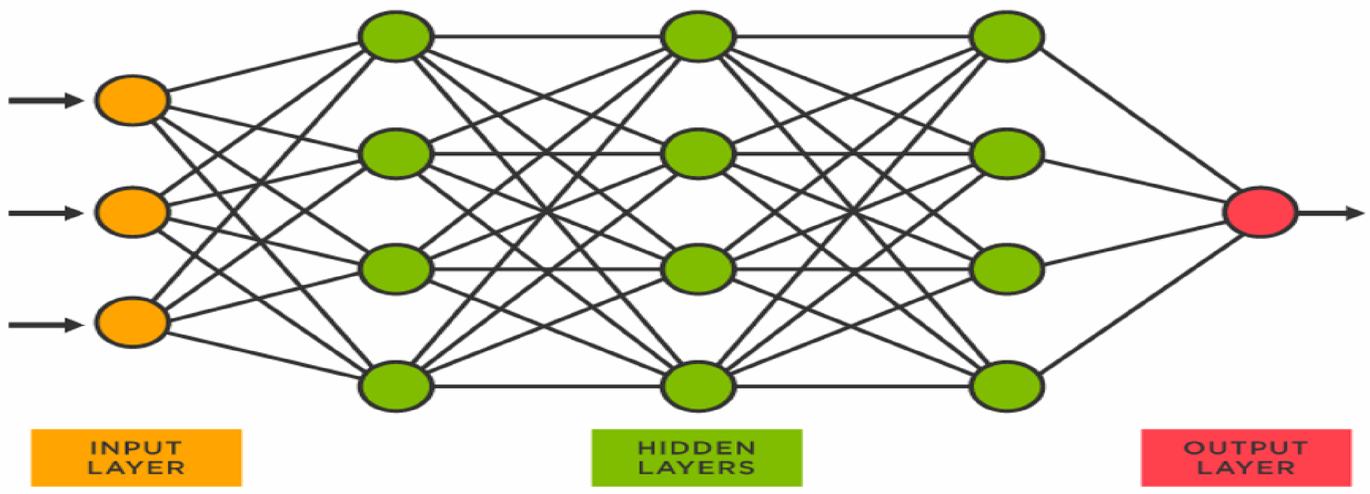
The main objective of the thesis is to enhance the analysis of time series and the reliability of neural networks in artificial intelligence (AI) applications through Topological Data Analysis (TDA). The thesis is structured around three core objectives and includes research stays at prestigious institutions to ensure a comprehensive and rigorous exploration.

2. Objectives

- **Short-term:** Extend the work by my advisors on persistent entropy in [1], to measure similarity between piecewise neural networks using ReLU activation. This metric can also serve as an early stopping method or loss function during training.
- **Mid-term:** Apply Topological Data Analysis (TDA) techniques to analyze time series data [2].
- **Long-term:** Integrate TDA into recurrent neural networks like LSTMs to enhance prediction accuracy, understand temporal dynamics, and explain network decisions. If successful and we have time, consider expanding to Transformers.

3. Artificial Intelligence: Neural Networks

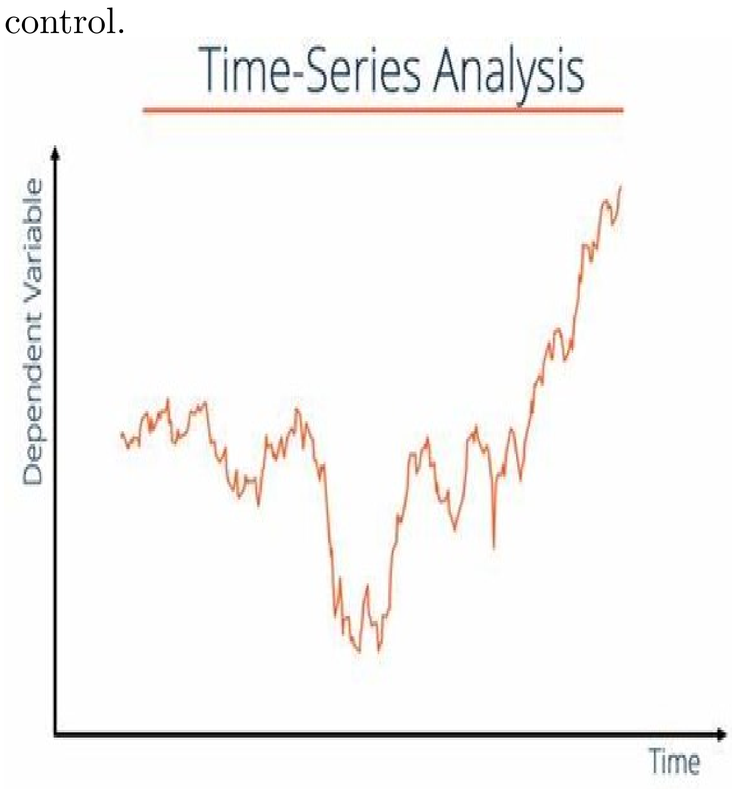
- **Artificial Intelligence:** Artificial intelligence (AI) is a type of computer technology which is concerned with making machines carry out work in an intelligent way, similar to how a human would. AI is at the forefront of technology transforming industries and tasks, from virtual assistants to advanced diagnostics.
- **Neural Networks:** A model inspired by the human brain neurons, used for regression and classification tasks.



- **Black Box Problem:** Lack of transparency in neural network [3] decision-making causing doubts of AI systems, especially in high-risk applications such as autonomous vehicles and medical diagnostics.

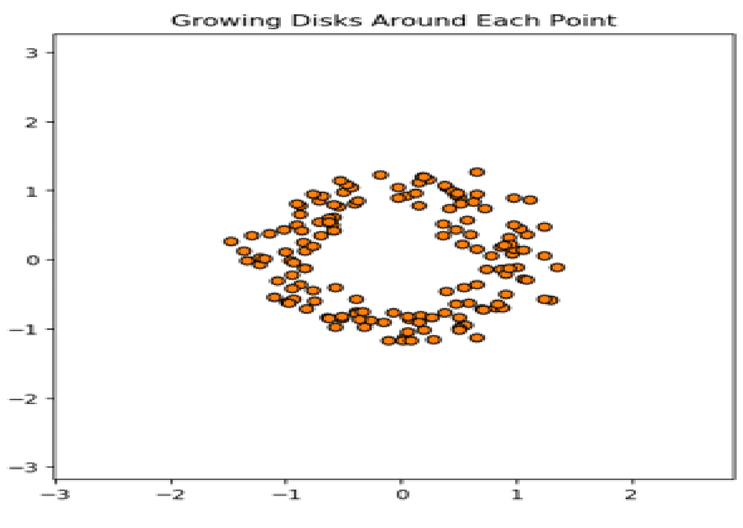
4. Time Series

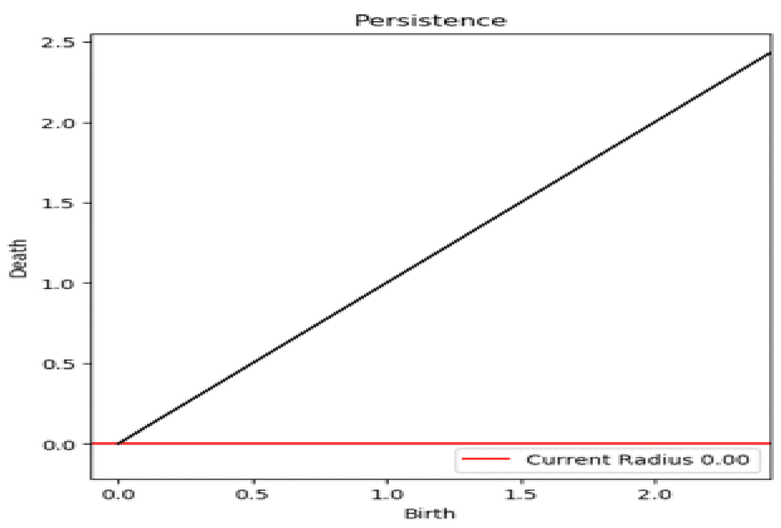
A time series is a sequence of N observations chronologically ordered on one (univariate) or several (multivariate) characteristics of an observable unit at different points in time. Time series are important in many situations like future predictions, pattern detection or quality control.



5. Topological Data Analysis

- **Topology** [4] studies properties of space that remain unchanged under continuous transformations, focusing on continuity, connectivity, and boundaries to understand spatial relationships and structures.
- **Topological Data Analysis** [5] applies topological and algebraic techniques to analyze data shapes, with applications in computer vision, shape description, and time series analysis.
- We use **persistent homology**, which examines topology across multiple scales, providing detailed insights into data structure and is represented by persistence diagrams, which show the persistence of topological features through their birth and death.





6. Planned Stays

- Institute of Electronic, Informatics and Telecommunications Engineering, at the Consiglio Nazionale delle Ricerche, Genoa, with Prof. Maurizio Mongelli, October 2024
- AIDOS Lab at Helmholtz Munich, with Prof. Bastian Grossenbacher Rieck, Summer 2025.

7. References

[1] Matteo Rucco, Rocio Gonzalez-Diaz, Maria-Jose Jimenez, Nieves Atienza, Cristina Cristalli, Enrico Concettoni, Andrea Ferrante, and Emanuela Merelli. A new topological entropy-based approach for measuring similarities among piecewise linear functions. *Signal Processing*, 134:130–138, 2017.

[2] Nalini Ravishanker and Renjie Chen. An introduction to persistent homology for time series. *WIREs Computational Statistics*, 13, 02 2021.

[3] Gary F. Marcus. Deep learning: A critical appraisal. *ArXiv*, abs/1801.00631, 2018.

[4] Herbert Edelsbrunner and John Harer. *Computational Topology: An Introduction*. Springer, 01 2010.

[5] Frédéric Chazal and Bertrand Michel. An introduction to topological data analysis: Fundamental and practical aspects for data scientists. *Frontiers in Artificial Intelligence*, 4, 10 2017.